



# Non perturbative renormalization in covariant light front dynamics

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# Non perturbative renormalization in covariant light front dynamics

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- State vector in a scalar model
- Covariant light-front dynamics
- Physical problem
- 2-particle problem
- 3-particle problem
- Generalization
- Including spin. . .

## State vector in a scalar model

Two scalar fields interacting as  $g\psi^2\varphi$ :

→ “Nucleon” field  $\psi$  of mass  $m$

→ “Pion” field  $\varphi$  of mass  $\mu$

A nucleon state vector obeys:

$$\hat{P}^2|p\rangle = M^2|p\rangle$$

in which the nucleon is **fully dressed** with pions.

Can  $|p\rangle$  be computed ?

# Covariant light-front dynamics

J. Carbonell, B. Desplanques, V.A. Karmanov, J.-F. Mathiot,  
Phys. Rep, **300** (1998) 215

- Why covariant?

$$t^+ = t + z \quad \Rightarrow \quad \sigma = \omega_\mu \cdot x^\mu, \quad \omega^2 = 0$$

Dependence on orientation of LF plane  
parametrized by 4-vector  $\omega^\mu$

- Light-front vacuum:

$$P_0 > 0 \quad \Leftrightarrow \quad P_+, P_- > 0$$

$\Rightarrow$  light-front vacuum is **trivial**

$\Rightarrow$  diagrams are not spoiled by vacuum bubbles

$\Rightarrow$  **Fock space** analysis

- Fock decomposition:

$$|p\rangle = \phi_1|N\rangle + \phi_2|N\pi\rangle + \phi_3|N\pi\pi\rangle + \dots$$

The  $\phi_i$  are the **wave functions**.

- Particles:

$\Rightarrow$   $\sigma$ -ordered diagrams

$\Rightarrow$  all particles are always **on-mass shell**

$\Rightarrow$  the triviality of the vacuum removes many diagrams

## Physical problem

$$\hat{P}^2 |p\rangle = M^2 |p\rangle, \quad \hat{P}_\mu = \hat{P}_\mu^0 + \hat{P}_\mu^{\text{int}}$$

$$\hat{P}_\mu^0 = \sum_i \int d^3\vec{k} \, d_i^\dagger(\vec{k}) d_i(\vec{k})$$

$$\hat{P}_\mu^{\text{int}} = \omega_\mu \int \frac{d\tau}{2\pi} \mathcal{H}_{\text{int}}(\tau\omega)$$

With  $[\omega \cdot \hat{P}_0, \mathcal{H}_{\text{int}}(\tau\omega)] = 0$  we get:

$$\left( \hat{P}_0^2 + 2(\omega \cdot p) \int \frac{d\tau}{2\pi} \mathcal{H}_{\text{int}}(\tau\omega) \right) |p\rangle = M^2 |p\rangle$$

$$\frac{1}{2\pi} \int \frac{d\tau}{\tau} \mathcal{H}_{\text{int}}(\tau\omega) | \mathcal{G}(p) \rangle = - | \mathcal{G}(p) \rangle$$

Fock components of  $| \mathcal{G}(p) \rangle$ :

$$\Gamma_n = (s - m^2) \phi_n = \begin{array}{c} \xrightarrow{p} \text{---} \bullet \text{---} \text{wavy lines} \end{array}^{n-1}$$

$\Gamma_n$

## 2-particle problem

$$|p\rangle = \phi_1 |N\rangle + \phi_2 |N\pi\rangle$$

$$\begin{array}{lcl}
 \Gamma_1 & & \Gamma_2 \\
 \text{---} \bullet \text{---} & = & \text{---} \bullet \text{---} \text{---} + \text{---} \bullet \otimes \text{---} \\
 \Gamma_2 & & \Gamma_1 \\
 \text{---} \bullet \text{---} & = & \text{---} \bullet \text{---} \text{---}
 \end{array}$$

$$\Rightarrow \Gamma_1 \text{---} \bullet \text{---} = \Gamma_1 \text{---} \bullet \text{---} \text{---} + \Gamma_1 \text{---} \bullet \otimes \text{---}$$

Trivially yields  $\delta m^2 = -g^2 \Sigma(p^2 = m^2)$ ,  
as identical to the perturbative case.

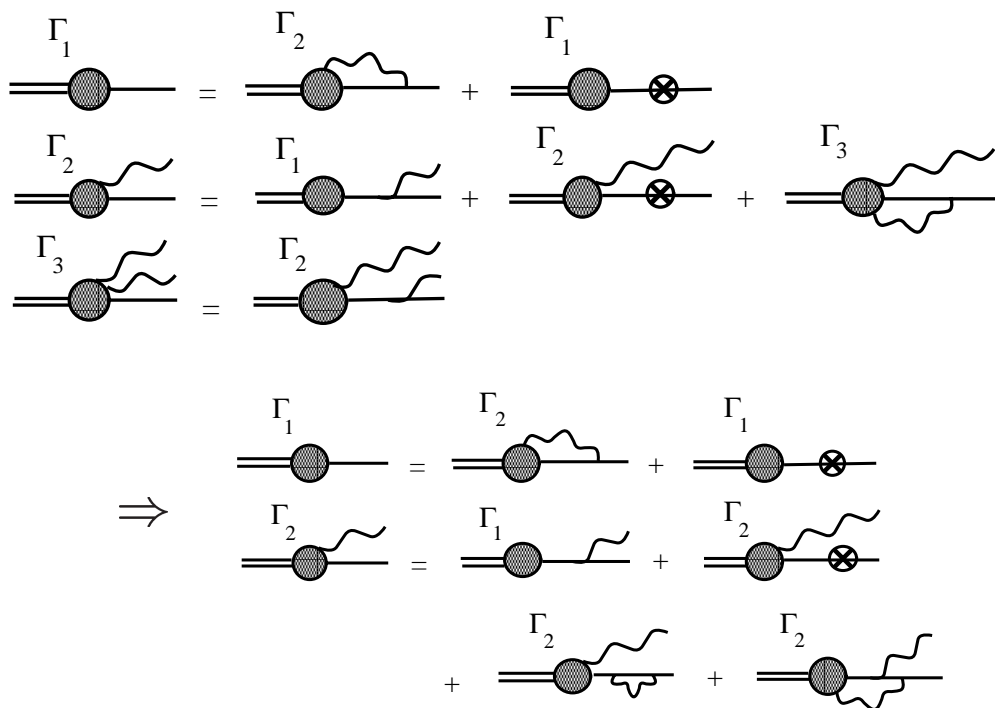
Iterating the relation:

$$\text{---} \text{---} \text{---} + \text{---} \text{---} \text{---} \text{---} + \dots + \text{---} \text{---} \text{---} \dots \text{---} \text{---} \text{---}$$

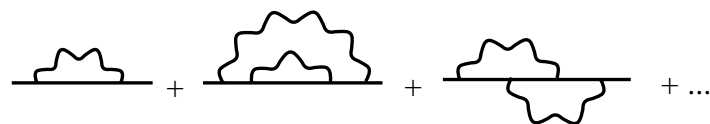
$\Rightarrow$  only one irreducible diagram

## 3-particle problem

$$|p\rangle = \phi_1 |N\rangle + \phi_2 |N\pi\rangle + \phi_3 |N\pi\pi\rangle$$



Iterating the relation:



$\Rightarrow$  infinite number of irreducible diagrams



Equation giving  $\Gamma_1$ :

$$\begin{aligned}
& \frac{\delta m^2 + g^2 \Sigma(s_1)}{(s - m^2)x_1} \Gamma_1(\vec{q}, \vec{n}) \\
& - \frac{g^2}{\delta m^2} \int \frac{d^3 \vec{q}'}{(2\pi)^3} \Sigma_i(\vec{q}', m^2) \Gamma_1(\vec{q}', \vec{n}) \\
& + g^2 \int \frac{d^3 \vec{q}'}{(2\pi)^3} \Sigma_i(\vec{q}', m^2) \Pi(\vec{q}', \vec{q}, \vec{n}, m^2) \Gamma_1(\vec{q}', \vec{n}) \\
& = \Gamma_1(\vec{q}, \vec{n}) \quad \equiv \lambda(\delta m^2) \Gamma_1(\vec{q}, \vec{n})
\end{aligned}$$

- $\delta m^2$  is determined by condition  $\lambda(\delta m^2) = 1$
- $\delta m^2$  differs from its perturbative value  $\delta m_0^2$

## Numerical solution

D. Bernard, T. Cousin, V.A. Karmanov, J.-F. Mathiot, Phys. Rev. D65, p. 025016 (2002)

- $m = 0.94, \mu = 0.14$
- $\alpha = \frac{g^2}{16\pi m^2}$
- Regularization:  $\|\vec{k}\| < L$

$L(\alpha = 3)$	5	10	50	100	200
$\frac{\delta m^2}{\delta m_0^2}$	1.052	1.040	1.025	1.022	1.020
$\langle q \rangle$	0.538	0.565	0.588	0.591	0.593

$\alpha(L = 200)$	1	10	100	1000
$\frac{\delta m^2}{\delta m_0^2}$	1.011	1.029	1.040	1.043

## Generalization

Difficulty *in principle* does not grow with  $n$ .

$$\begin{aligned}
 \Gamma_1 &= \Gamma_2 + \Gamma_1 \\
 \vdots & \\
 \Gamma_{n-1} &= \Gamma_{n-2} + \Gamma_{n-1} + \Gamma_n \\
 \Gamma_n &= \Gamma_{n-1}
 \end{aligned}$$

$\Rightarrow n \rightarrow n - 1$  is always trivial

$\Rightarrow$  iteration  $i \rightarrow i - 1$  can be made systematically

$\Rightarrow$  going from  $n$  to  $n + 1$  requires to restart computation

## Including spin...

Two spinor fields exchanging a scalar:  $g\bar{\psi}\varphi\psi$

$\Rightarrow$  Same structure for the 3-body system with **3 additional contact terms**:

$$\begin{aligned}
 g - g & : \quad \text{---} \text{wavy} \text{---} \times \text{---} \text{wavy} \text{---} = g^2 \left( -\frac{\not{p}}{2\omega.p} \right) \\
 g - \delta m & : \quad \text{---} \text{wavy} \text{---} \times \text{---} \otimes \text{---} = g \delta m \left( -\frac{\not{p}}{2\omega.p} \right) \\
 \delta m - \delta m & : \quad \text{---} \otimes \text{---} \times \text{---} \otimes \text{---} = \delta m^2 \left( -\frac{\not{p}}{2\omega.p} \right)
 \end{aligned}$$

$\Rightarrow$  Same difficulty *in principle* as scalar case, with **7 wavefunctions** instead of 3

## Conclusion

- heavy use of **covariant** LF special features
- “satisfactory” for 2- and 3-body cases
- **iterative method** should allow for generalization to  $n$ -body case
- computational complexity stays under control, i.e. does not explode for high  $n$
- extended to more physical models... ?